PHYS1112 Final Exam

Formulae and Constants

Constants:

Acceleration due to gravity $g = 9.80$ m/s^2.

Gravitational constant $G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2

Avogadro's number $N_A = 6.022 \times 10^{23}$ molecules/mol

Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K

Gas constant $R = 8.31$ J/mol·K

Atmospheric pressure, 1 atm $= 1.013 \times 10^5$ Pa

Calories: 1 cal $= 4.186$ J

Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ W·m^{-2}·K^{-4}

Speed of sound in air at room temperature $= 340$ m/s

Astronomical Data:

Body	Mass (kg)	Radius (m)	Radius of its orbital motion (m) (center-to-center)
Sun	1.99×10^{30}	6.96×10^{8}	—
Earth	5.97×10^{24}	6.37×10^{6}	1.50×10^{11}
Moon	7.35×10^{22}	1.74×10^{6}	3.84×10^{8}

Formulae

Constant acceleration motions:

Straight-line motion

$$a_x = \text{constant}$$

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$$

Fixed-axis rotation

$$\alpha_z = \text{constant}$$

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$$

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$$

Center of Gravity: $\vec{r}_{\text{cm}} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}$

$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}, \quad y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i}, \quad z_{\text{cm}} = \frac{\sum m_i z_i}{\sum m_i}$$

Rotational speed: $v_{\text{tan}} = \omega r$

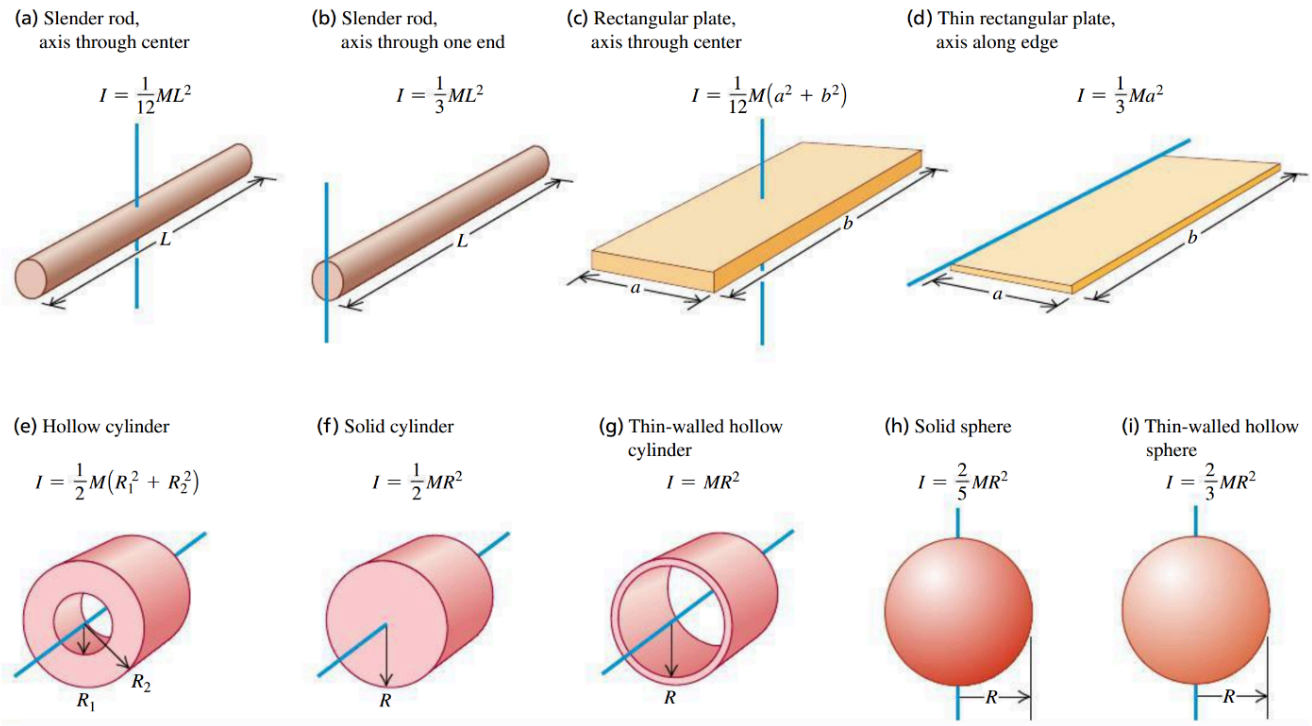
Tangential acceleration in rotation: $a_{\text{tan}} = \dfrac{dv_{\text{tan}}}{dt} = \alpha r$

Radial acceleration in rotation: $a_{\text{rad}} = v^2/r = \omega^2 r$

Moment of inertia: $I = \sum m_i r_i^2 \quad \longrightarrow \quad \int r^2\, dm = \int r^2 \rho\, dV$

Parallel axis theorem: $I_p = I_{\text{cm}} + Md^2$

Moments of inertia of various bodies:



Kepler's third law: $T = 2\pi a^{3/2}/\sqrt{Gm_S}$

Simple harmonic motion: $x = A\cos(\omega t + \phi)$, where

$$A = \sqrt{x_0^2 + \frac{v_{0x}^2}{\omega^2}} \quad \text{and} \quad \phi = \begin{cases} \tan^{-1}\left(-\dfrac{v_{0x}}{\omega x_0}\right) & \text{if } x_0 > 0; \\ \tan^{-1}\left(-\dfrac{v_{0x}}{\omega x_0}\right) + \pi & \text{if } x_0 < 0. \end{cases}$$

Simple harmonic oscillator:

$$\omega = \begin{cases} \sqrt{k/m} & \text{(block-spring)} \\ \sqrt{g/L} & \text{(simple pendulum)} \\ \sqrt{mgd/I} & \text{(physical pendulum)} \end{cases}$$

Damped harmonic oscillator: $x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega' t + \phi)$, $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$.

Forced harmonic oscillations: $A = \dfrac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$.

Wave equation: $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$.

Power propagation in wave motion: $P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$.

Wave speed on a string: $v = \sqrt{F/\mu}$.

Doppler effect: $f_{\mathrm{L}} = \left(\dfrac{v + v_{\mathrm{L}}}{v + v_{\mathrm{S}}}\right) f_{\mathrm{S}}$

Fahrenheit scale: $T_{\mathrm{F}} = (9/5)T_{\mathrm{C}} + 32$

Linear expansion: $\Delta L/L_0 = \alpha\Delta T$; Volume expansion: $\Delta V/V_0 = \beta\Delta T$.

Rate of heat flow by conduction: $H = kA(T_H - T_C)/L \longrightarrow$ continuum limit: $H = -kA\dfrac{dT}{dx}$.

Stefan-Boltzmann Law: $H = Ae\sigma T^4$.

Maxwell-Boltzmann distribution: $f(v) = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-mv^2/2kT}$,

$$v_{\mathrm{av}} = \sqrt{\frac{8kT}{\pi m}}, \quad v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}}.$$

Heat capacity ratio: $\gamma = C_P/C_V$.

Adiabatic processes for an ideal gas:

$$PV^\gamma = \text{constant}, \quad TV^{\gamma-1} = \text{constant}, \quad \text{work done } W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$$

Trigonometric Relations:

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$$
$$\sin(2A) = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\sin^2\frac{A}{2} = \frac{1}{2}(1-\cos A)$$
$$\cos^2\frac{A}{2} = \frac{1}{2}(1+\cos A)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A}$$

Series Expansion:

$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \cdots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$
$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots, \quad |x| < \pi$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, |x| < 1$$

Integrals:

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \quad \left|\tan^{-1}\left(\frac{x}{a}\right)\right| < \frac{\pi}{2}$$
$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln\left(a^2+x^2\right)$$
$$\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$$
$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$
$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx}$$
$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$$
$$\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$
$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$
$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$
$$\int \tan x\,dx = -\ln|\cos x|$$
$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\pi/a}$$
$$\int x\ln(x)\,dx = x\ln(x) - x$$